A METHOD OF DETERMINING TEMPERATURE-DEPENDENT THERMAL CONDUCTIVITY

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Inzhenerno-Fizicheskii Zhurnal, Vol. 10, No. 4, pp. 513–515, 1966

UDC 536.2

A method is described for determining thermal conductivity, based on solution of the nonlinear equations of heat conduction, for a quasi-steady thermal state in the presence of a moving boundary. A formula is obtained for evaluating the time to establish the quasi-steady state with a given accuracy. One possible experimental means of realizing the moving boundary is suggested.

At present only one method is known for determining thermal conductivity, based on exact solution of the nonlinear heat conduction equation [1]. The problem has been solved for a semi-infinite body whose surface temperature is kept constant.

We shall show that a solution also exists for other boundary conditions, in particular, for the case of a plane, constant-temperature front moving at constant speed within a semi-infinite body. The initial temperature of the body is assumed constant.

Under these conditions a quasi-steady thermal state is established in the body [2], such that the equality

$$\frac{\partial t}{\partial \tau} = -v\frac{\partial t}{\partial x} \tag{1}$$

is valid. The nonlinear heat conduction equation may be written in the form [3]

$$\frac{d}{dt}\left[\lambda(t)\frac{\partial t}{\partial x}\right] - \alpha c(t)\gamma(t) = 0, \tag{2}$$

where

$$\alpha = \frac{\partial t}{\partial \tau} \Big/ \frac{\partial t}{\partial x}. \tag{3}$$

In the conditions of this problem $\alpha = -v$. Integrating (2), taking into account that when $x \to \infty$ we have $t \to t_0$, $\partial t/\partial x \to 0$, and using (1), we obtain

$$\lambda(t) = \frac{v^2 \int\limits_{t_0}^{t} c(t)\gamma(t)\,dt}{\partial t/\partial \tau}. \tag{4}$$

In the experiment it is necessary to determine the gradient of temperature with time at one point of the specimen. We must also know the specific heat and the specific weight as functions of temperature. If $\gamma = \text{const}$, it is sufficient to know the mean specific heat over the temperature range concerned.

In order to estimate the time to establish the quasi-steady state, we shall employ the solution to the problem of the temperature field in a semi-infinite body in the presence of a constant-temperature plane moving with constant velocity inside the body, under the assumption that the thermophysical properties do not depend on temperature [4]:

$$\frac{t-t_0}{t_1-t_0} = \frac{1}{2}\left[\exp\left(-\frac{v}{a}\zeta\right)\cdot \operatorname{erfc}\frac{\zeta - v\tau}{2\sqrt{a\tau}} + \operatorname{erfc}\frac{\zeta+v\tau}{2\sqrt{a\tau}}\right]. \tag{5}$$

Here ζ is the distance from the moving boundary to the the point with coordinate x, $\zeta = x - v\tau$.

When $\tau \to \infty$, Eq. (5) transforms to the well-known equation of a quasi-steady state in the presence of moving sources [2]

$$\frac{t-t_0}{t_1-t_0} = \exp\left(-\frac{v}{a}\zeta\right). \tag{6}$$

We put

$$\operatorname{erfc}\frac{v\tau-\zeta}{2\sqrt{a\tau}} \leqslant \varepsilon,$$

where ε is a small positive number whose value is chosen in terms of the required accuracy. Then

$$\operatorname{erfc}\frac{\zeta - v\tau}{2\sqrt{a\tau}} \geqslant 2-\varepsilon, \quad \operatorname{erfc}\frac{\zeta+v\tau}{2\sqrt{a\tau}} < \varepsilon,$$

and the quantity $(t - t_0)/(t_1 - t_0)$ differs from the steady value by less than $\varepsilon/2$.

We designate

$$K_\varepsilon = (v\tau^* - \zeta)/2\sqrt{a\tau^*} \tag{7}$$

when

$$\operatorname{erfc}\left[(v\tau^* - \zeta)/2\sqrt{a\tau^*}\right] = \varepsilon. \tag{8}$$

From (7) we obtain a formula for calcuating the time to establish quasi-steady state with a given accuracy for any ζ

$$\tau^* = \left(\frac{v\zeta}{a} + 2K_\varepsilon^2 + 2K_\varepsilon\sqrt{\frac{v\zeta}{a} + K_\varepsilon^2}\right)\frac{a}{v^2}. \tag{9}$$

The moving boundary may be realized by melting, sublimation, grinding, etc., of the surface of the material under examination.

One method of realizing a moving boundary experimentally is to use thermomechanical destruction of the surface by means of friction of the material against a moving grooved plate of siliconized graphite, heated to a high temperature [5]. Heating is accomplished by radiation from a heater positioned parallel to the free surface of the plate.

NOTATION

t—temperature; t_0—initial temperature; τ—time; τ^*—time to establish quasi-steady state; x—coordinate; ζ—distance from moving boundary; λ—thermal conductivity; c—specific heat; a—thermal diffusivity; γ—specific weight; v—velocity of moving boundary.

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25 August 1964